

Theoretical predictions of residue cross-sections for the superheavy elements

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1. Introduction

Productions of the superheavy elements are predicted with the two-step model for fusion of massive systems [1], combined with the theory of the statistical decay of the compound nucleus. As is well known, the fusion of lighter heavy-ion systems is determined by whether the incident system can enter the inner side of the Coulomb barrier or not, but in heavy systems an additional process is indispensable for the formation of the compound nucleus, because there is a conditional saddle point which locates between the position of the Coulomb barrier top and the spherical shape of the compound system. Therefore, the fusion of massive heavy-ion systems which are necessary for the synthesis of superheavy elements (SHE) requires two steps as schematically shown in the first figure; firstly the approaching phase up to the contact of the incident ions after overcoming the Coulomb barrier and secondly the shape evolution phase to the spherical compound nucleus, starting from the pear-shaped composite system made by the projectile and the target of the incident channel. Thus, the product of the sticking and the formation probabilities (P_{stick} and P_{form}), which are obtained by solving dynamics in the two steps, respectively, gives the fusion probability P_{fusion} .

$$P_{\text{fusion}}^J(E_{\text{c.m.}}) = P_{\text{stick}}^J(E_{\text{c.m.}}) \times P_{\text{form}}^J(E_{\text{c.m.}}) \quad (1)$$

where J denotes a total spin of the system and $E_{\text{c.m.}}$ a c.m. incident energy. The amalgamated system is expected to be excited internally, that is, the incident kinetic energy is transferred to a thermal energy. The dissipation of the incident kinetic energy may start before the top of the Coulomb barrier, or may start at the moment of touching of the two nuclear matters. If we presume the latter case, the sticking probability is given by a quantum-mechanical barrier penetration factor or simply by a step function at the barrier height, and the formation probability should be calculated by a Langevin equation with the initial momentum being defined by the incident kinetic energy and with a time-dependent temperature which describes a heating-up process. On the other hand, if we presume the former case, the sticking probability should be calculated by a Langevin equation with a frictional force and with a time-dependent temperature, and the formation probability is calculated with a Langevin equation with a constant temperature for shape evolution, if the incident energy is mostly damped at the moment of the contact or of the amalgamation. In view of the results of the Deep-Inelastic Collisions (DIC), it is natural to presume the former. Of course, one would be interested in the enhancement in the sub-barrier energy which is known to originate from coherent couplings with some other channels and to be very important in lighter heavy-ion systems. But, taking into account a strong fusion hindrance which is well known to exist in massive systems, an incoherent treatment, i.e., a treatment of effects of the couplings as a friction would be reasonable.

2. Approaching Phase

We employ a classical treatment for the description of the relative motion of the incident ion system with a frictional force and an associated fluctuation force which is missing in the original classical treatments [2, 3]. The equation is as follows,

$$\begin{cases} m \frac{d^2 r}{dt^2} = -\frac{\partial V}{\partial r} - \frac{\partial}{\partial r} \frac{\hbar^2 L^2}{2\mu r^2} - C_r(r) \frac{dr}{dt} + R_r(t) \\ \frac{dL}{dt} = -\frac{C_T(r)}{\mu} \left[L - \frac{5}{7} L_0 \right] + R_T(t) \end{cases} \quad (2)$$

$$\langle R_i(t) R_j(t') \rangle = 2T(t) \cdot C_i(r(t)) \cdot \delta_{ij} \cdot \delta(t-t')$$

where m is the reduced masse and V is the sum of the Coulomb and the nuclear attractive potentials. $C_i(r)$ is the radial and tangential frictions, respectively, where the rolling friction is neglected. L_0 is the incident angular momentum and $5/7 L_0$ so called sliding limit. $R(t)$ denotes a Gaussian random force with zero mean value. The last equation is the dissipation-fluctuation theorem assumed and in case $i=j=r$, r^2 factor is necessary for $C_i(r)$. As for the friction in the approaching phase, there are two models available; one is the surface friction model (SFM) [2] and the other the proximity friction [4]. The former is rather successful in reproducing DIC data except spins of outgoing fragments which appears to require the inclusion of the rolling friction. And the strengths are very different between the radial and tangential frictional forces, which may be unnatural. On the other hand, the latter does not take into account effects of strong couplings to inelastic channels which are important in low energy. Thus, no outstandingly good model for the friction in the approaching phase is available for the moment. Thus, these models were used to calculate sticking probabilities [5]. It turned out that SFM is much stronger than the proximity one, i.e., sticking probabilities calculated by SFM are extremely small, though which one is realistic is not determined yet. At the same time, there is a common feature that the radial momentum has a distribution with a Gaussian shape which would be due to the assumption of the Gaussian random forces associated with the frictional force. The width of the distribution is consistent with the temperature determined by the internal energy transferred from the kinetic energy. The distribution is used for the initial condition for the dynamics of the second phase, i.e., for shape evolution toward the spherical shape. In this sense the model should be called a stochastic two-step model.

3. Shape Evolution

As stated in the Introduction, the amalgamated system locates at the outside of the conditional saddle point or at the outside of the ridgeline. Therefore, in order to obtain the formation probability, we have to solve shape evolution towards the spherical shape under a frictional force and its associated random force. The ratio between the number of trajectories that pass over the ridge line and the total number gives the formation probability, though most of them return back to reseparation due to the conservative potential calculated with the liquid drop model (LDM). An example of LDM energy surface is shown in the second figure for the system with $A=272$ and $Z=112$. Shapes are specified in terms of the two-center parameterization $(R/R_0, a)$ with the neck parameter fixed to be 1.0 [6]. The cross in the figure denotes the contact configuration of $^{64}\text{Ni}+^{208}\text{Pb}$, i.e., the starting point of the evolution.

The dynamics is again described by a Langevin equation, generally multi-dimensional one,

$$\begin{cases} \frac{dq_i}{dt} = (m^{-1})_{ij} p_j \\ \frac{dp_i}{dt} = -\frac{\partial V^J}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t) \\ \langle g_{ik} g_{jk} \rangle = \gamma_{ij} T^J \end{cases} \quad (3)$$

where V denotes the LDM potential plus the centrifugal force for spin J . The last equation is again the dissipation-fluctuation theorem with a constant temperature T^J for spin J . We calculate the friction tensor by employing so-called one-body model (OBM), i.e., one-body wall-and-window formula [7] with the two-center parameterization of nuclear shapes. For a given initial momentum conjugate with the distance, we calculate many trajectories, some of which pass over the ridgeline. Then, the formation probability is given by the average over the initial momentum distribution obtained in the first step.

4. Residue Cross Sections

Assuming the compound nucleus theory of reactions, residue cross sections of the superheavy elements are given by a product of the fusion probability P_{fusion} and the survival probability P_{surv} as follows,

$$\sigma_{\text{res}} = \pi \hbar^2 \sum (2J+1) P'_{\text{fusion}}(E_{\text{c.m.}}) P'_{\text{surv}}(E^*) \quad (4)$$

where $E^* = E_{\text{c.m.}} + Q$ with the fusion Q -value. P_{surv} denotes the probability for the compound nucleus to survive against fission and charged particle emission. It is calculated by the new computer program which is constructed, based on the theory of the time-dependent statistical decay [8]. An essential parameter is the shell correction energies for the superheavy elements. Although there are many predictions on them by the structure calculations with various levels of nuclear model [9], but unfortunately they differ with each other. We introduce a single reduction factor 0.4 for P . Moeller et al's predictions, because their predictions mostly appear to be largest compared with others' in the absolute values for nucleides around $Z=114$. An example of the residue cross section is shown in the last figure for $^{64}\text{Ni} + ^{208}\text{Pb}$ system leading to $Z=112$ element after one neutron emission. The result is compared with the available data [10]. Theory reproduces the data remarkably well. Similar results are obtained for $^{70}\text{Zn} + ^{208}\text{Pb}$, though the data to be compared are few. Predictions are made on $Z=113$ and 114 elements.

It should be worth to remind that the model has been already applied to the hot fusion path, i.e., $^{48}\text{Ca} + \text{actinide}$ systems with the same parameters and resulted in a reasonably good reproduction of the available data [1, 11].

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